Relative Performance Evaluation and Limited Liability

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Abstract

We analyze the role of relative performance evaluation when a principal has several agents, who face correlated shocks. If limited liability constraints are binding, relative performance evaluation may be of no value if the principal is restricted to symmetric contracts. However, with asymmetric contracts, where agents are induced to choose different effort levels, relative performance measures can be used in order to reduce informational rents. Relative performance evaluation is a way of reducing the rents of the high effort agent, who will in general be worse off than the low effort agent.

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1 Introduction

Let us begin with an empirical puzzle — why are relative performance measures are used so infrequently in agrarian contracts? There is a large literature on the nature of contracts between landlords and tenant, but this literature suggests that rental obligations are conditioned only on the output of the tenant, and only infrequently, and in exceptional circumstances, upon output realizations of other tenants in the village. The empirical evidence also suggests that output shocks are, to some extent, common across farmers in contiguous locations. The standard model of contracting under moral hazard (e.g. Holmstrom, 1979) suggests that relative performance measures can provide more powerful incentives and greater insurance to tenant farmers. Thus the absence of relative performance measures is a puzzle.

One response to this puzzle is to say that shocks are idiosyncratic and independent across agents. However, if the production technology is characterized by constant returns to scale, I claim that one can always partition plots of land among tenants so that shocks are highly correlated. Take the standard model of moral hazard. Assume that expected output is homogeneous of degree one in land, K, and in effort e (which is the same as aggregate labour input). Let the plot of land be of unit size. Output is random and takes values in the set $\{H, L\}$, where the probability of H being realized is given by $p(e)$, which is an increasing function of e, the agent's effort. Suppose now that the plot of land is divisible, into N parts, where the effort on each part is e/N. Consider first the possibility that the risks on these N plots are independent. If the outcomes are binary, on each of these N parts, then this implies that the aggregate output is the sum of these random variables on the component parts. As the number of parts, N, tends to infinity, we deduce that aggregate output must be deterministic, by the law of large numbers. Hence a model where output is random at the level of the aggregate plot is inconsistent with independent risks. Indeed, this argument shows that the only risk must be aggregate.

Suppose now that risk is aggregate, and suppose also that there are several landlords and several tenants in the village. The question then is, why does not each landlord not divide up his land between two tenants or more tenants? By doing so, he could use relative performance measures, i.e. he could condition the payments one tenant receives also on the realized outcome of the other tenant. Hence we are back at our initial puzzle, i.e. why are such arrangements not common?

In this paper, we evaluate the role of relative performance measures in a situation of limited liability. Limited liability is a plausible assumption in poor countries, and models of contracting with limited liability have been increasingly used in the development economics literature — see, for example, Banerjee and Newman (1993) or Mookherjee and Ray (2002). We find that the role of relative performance measures is qualitatively different when agents have limited liability, in contrast with their role under unlimited liability. if a principal must offer the same contract to both agents (the case of *symmetric contracts*), then limited liability constraints can make relative performance evaluation valueless. On the other hand, if the principal seeks to implement different effort levels for two agents (or if two different principals induce different effort levels, (the case of *asymmetric contracts),* then relative performance is valuable in providing incentives for the high effort agent. Intuitively, relative performance evaluation reduces the amount of rent that the principal has to provide for the high effort agent. Since the high effort agent may consequently be worse off than the low effort agent, considerations of equity plus limited liability may provide one explanation for the absence of relative performance measures in practice.

2 The Model: Symmetric Contracts

I consider a model with one risk neutral principal who owns two plots of land, each of size one, who contracts with two identical agents (alternatively, the model can also be interpreted as one with two agents, who work for two identical principals). The agent must choose effort $e \in [0, 1]$, where the cost of effort is $d(e)$, where d is increasing, differentiable and convex. This gives rise to a realization of output $y \in \{H, L\}$. Assume that the shocks are perfectly correlated. Specifically, the probability distribution on outputs conditional on $\mathbf{e} = (q, p)$ (i.e. when the row agent chooses effort q and the column agent chooses effort p, $q \leq p$, is given by:

	H	L
H	q	0
L	$p-q$	$1-p$

A contract for an agent consists of a four-tuple $\mathbf{w} = (w_{HH}, w_{HL}, w_{LH}, w_{LL})$. Assume that the agent's utility is given by the expectation of $u(w)$ minus the cost of effort, where u is strictly concave.

Suppose the principal wants to implement the effort level e for agent i, given that agent j chooses effort level $\bar{e}$. Consider first the case where $e \leq \bar{e}$ — this subsumes the case of a symmetric equilibrum where both agents choose the same effort level. The payoff for agent i from choosing effort level $e' \leq \bar{e}$ is given by

$$e'u(w_{HH}) + (\bar{e} - e')u(w_{LH}) + (1 - \bar{e})u(w_{LL}) - d(e') \tag{1}$$

This gives rise to an incentive compatibility constraint

$$u(w_{HH}) - u(w_{LH}) \geq d'(e) \tag{IC}$$

where IC must hold with equality if $e < \bar{e}$. There is also an "upward" incentive constraint, which requires that the agent does not choose an effort level greater than $\bar{e}$, but it is easily satisfied provided that the principal chooses $w_{HL} \leq w_{LL}$, since effort is costly.

The participation constraint for an agent is given by

$$eu(w_{HH}) + (1 - e)u(w_{LL}) - d(e) \geq \bar{u} \tag{PC}$$

where $\bar{u}$ denotes the utility of the agent's outside option. Finally, we have the limited liability constraints, $\mathbf{w} \geq \mathbf{0}$.

Since the principal is risk neutral, his problem is to minimize expected wage payments subject to the constraints IC, PC and the limited liability constraint. Expected wage payments are given by

$$ew_{HH} + (1 - e)w_{LL} \tag{EW}$$

As we have already noted, w_{HL} enters only the incentive constraint in the upward direction and does not enter either the objective nor the other constraints, and hence may be set arbitrarily to any number less than w_{LL}. Furthermore, since w_{LH} does not enter the participation constraint, it is clear that it it optimal to set it to zero, since that relaxes the incentive constraint IC maximally.

Let w^*_{HH} be the value of the wage which solves $u(w^*_{HH}) = d'(e)$. If $w_{HH} \geq w^*_{HH}$, then IC will be satisfied. Let w^*_{LL} be the value fo the wage which solves PC with equality when $w_{HH} \geq w^*_{HH}$,i.e. $u(w^*_{LL}) = \frac{\bar{u}+d(e)-ed'(e)}{(1-e)}$. We have three possibilities:

a) If $w^*_{LL} \in (0, w^*_{HH}]$, then the optimal contract involves setting $w_{HH} = w^*_{HH}, w_{LL} = w^*_{LL}$ (recall that $w_{LH} = 0$ and w_{HL} can also be set to zero). In this case, IC and PC both bind. Since $w_{LL} \neq w_{LH}$, one has relative performance evaluation, and this reduces the cost to the principal of satisfying PC.

b) If $w^*_{LL} > w^*_{HH}$, the optimal contract has $w_{HH} = w_{LL} = \bar{w}$, where $\bar{w} = u^{-1}(\bar{u}+d(e))$. Again one has relative performance evaluation, and in this case the principal perfectly insures the agent since the incentive constraint IC does not bind.

c) If $w^*_{LL} \leq 0$, then the optimal contract has $w_{HH} = w^*_{HH}, w_{LL} = 0$, since a negative value would violate the limited liability constraint. Since $w_{LL} = w_{LH}$, there is no relative performance evaluation in the optimal contract.

Proposition 1 *Let $e \leq \bar{e}$. Suppose that the agent receives rents in the single agent case at effort level e. Then there is no relative performance evaluation for this agent in the two agent case, i.e. $w_{LH} = w_{LL}$*

w_{LH} must be set to zero (from IC)

w_{LL} affects PC but not IC

If PC is not binding and $w_{LL} > 0$, principal can reduce w_{LL}.

Conversely, if the agent gets no rent and limited liability constraints are not binding, then there must be relative performance evaluation.

To summarize, we find that a relative performance measure is used if and only if the agent receives no rents from the relationship, i.e. if the participation constraint binds. If PC does not bind and the agent receives positive rents, the principal does not benefit from conditioning the payment to agent i upon the output realization of the other agent, j. It is true that relative performance is informative in this case. Indeed, the principal can insure the agent perfectly without affecting incentives by setting $w_{LH} = 0, w_{LL} = w^*_{HH}$. However, this only raises the expected payments of the principal without providing any benefit, and hence is not optimal from the principal's point of view.

2.1 The Value of Information

Our analysis of relative performance evaluation is related to the value of information to the principal in the context of limited liability. Consider the situation where the agent chooses effort e or e', $e' < e$, and this induces the following probability distribution on the set of signals $\{H, L, L'\}$.

	H	L	L'
e	e	$1-e$	0
e'	e'	$1-e$	e'

Assume that principal's information partition is such that he costlessly observes whether the event H or the event $\{L, L'\}$ occurs. Furthermore, in the event that $\{L, L'\}$ materializes, he can find out which of these two signal was realized, by incurring a small cost (possibly zero). The question is, under what conditions does the principal have an incentive to acquire this additional information.

If the agent has unlimited liability and is risk averse, the principal will have a strong incentive to acquire this additional information, and the contract will hence be contingent on the most finest information partition, provided that the cost of information acquisition is sufficiently small. Indeed, in this case, by acquiring this information the principal can ensure the first best — he can fully insure the agent, while providing sufficient incentives for effort. However, if limited liability constraints are binding, additional information may be valueless. If the principal cannot reduce the wage payments when L' materializes below some lower bound, say 0, then it may be optimal to offer 0 after both L and L', while providing incentives through a high wage in the event of H. In this case, additonal information is valueless.

3 Asymmetric Contracts

We now consider the more general possibility, where the two agents can be offered different contracts. Suppose that agent j is choosing effort level $\bar{e}$, and suppose that the principal seeks to induce effort level $e^* > \bar{e}$ for agent i. We now show that relative performance evaluation can be effective in providing incentives at low cost, for effort levels which are relatively high. The reason for this is that the marginal return to effort for the agent is $w_{HL} - w_{LL}$, (rather than $w_{HH} - w_{LH}$, as in the previous case). The first order condition for an optimum is

$$u(w_{HL}) - u(w_{LL}) = d'(e^*) \tag{LIC}$$

In addition, if $w_{HH} < w_{HL}$, the first order condition is not sufficient, since a global incentive constraint must be satisfied. Let $\hat{e}$ be the effort level which solves

$$u(w_{HH}) - u(w_{LH}) = d'(\hat{e}) \tag{2}$$

Global optimality of e^* requires that

$$(\bar{e} - \hat{e})[u(w_{HH}) - u(w_{LH})] + (e^* - \bar{e})[u(w_{HL}) - u(w_{LL})] \geq d(e^*) - d(\hat{e}) \tag{GIC}$$

The participation constraint is

$$\bar{e}u(w_{HH}) + (e^* - \bar{e})u(w_{HL}) + (1 - \bar{e})u(w_{LL}) - d(e) \geq \bar{u} \tag{PC'}$$

Hence the principal seeks to minimize expected wage payments, as given by

$$\bar{e}w_{HH} + (e^* - \bar{e})w_{HL} + (1 - \bar{e})w_{LL} \tag{3}$$

subject to the local incentive constraint LIC, the global incentive constraint GIC, the participation constraint PC', and the limited liability constraints.

We now show that the cost of effort to the principal is kinked at $e = \bar{e}$, with a right hand derivative which is strictly below the left hand derivative. Note that at $e \leq \bar{e}$, the cost of effort is given by the participation constraint and limited liability constraints as:

$$C(e) = ew_{HH} = ev[d'(e)] \tag{4}$$

where $v(.)$ is the inverse of the agent's utility function $u(.)$. Furthermore, we also have that $w_{LL} = w_{LH} = 0$, with w_{HL} set arbitrarily at some value less than w_{HH}.

$$C'(e) = ew_{HH} = v[d'(e)] + \frac{ed''(e)}{u'(w_{HH})} \tag{5}$$

Suppose now that the principal seeks to implement an effort level e^* which is slightly greater than $\bar{e}$. Note first that the principal can do this by changing w_{HL} alone so that $u(w_{HL}) = d'(e^*)$, while leaving all other contingent wage payments unaltered. In particular, w_{HH} need not change. Hence the cost of implementing e^* must be less than the cost of following this scheme, i.e.

$$C(e^*) \leq \bar{e}v[d'(\bar{e})] + (e^* - \bar{e})v[d'(e^*)] \tag{6}$$

Indeed the principal can do strictly better. Notice that the global incentive constraint holds as a strict inequality under the above contract, as does the participation constraint. Therefore the can principal reduce w_{HH} as well until GIC bites. I.e. he chooses w_{HH} to solve

$$(\bar{e} - \hat{e})u(w_{HH}) + d(\hat{e}) + d'(e^*)(e^* - \bar{e}) - d(e^*) = 0 \tag{7}$$

Intuitively, by choosing $w_{HL} = d'(e^*)$, the principal pays a marginal informational rent which equals the difference between $d'(e^*)(e^* - \bar{e})$ and the cost of the additional effort, $d(e^*) - d(\bar{e})$. Since the cost of effort function is convex, this difference is strictly positive. This marginal informational rent can be used to reduce the payments on previous effort levels, i.e. w_{HH} can be chosen to be less than $d'(\bar{e})$.

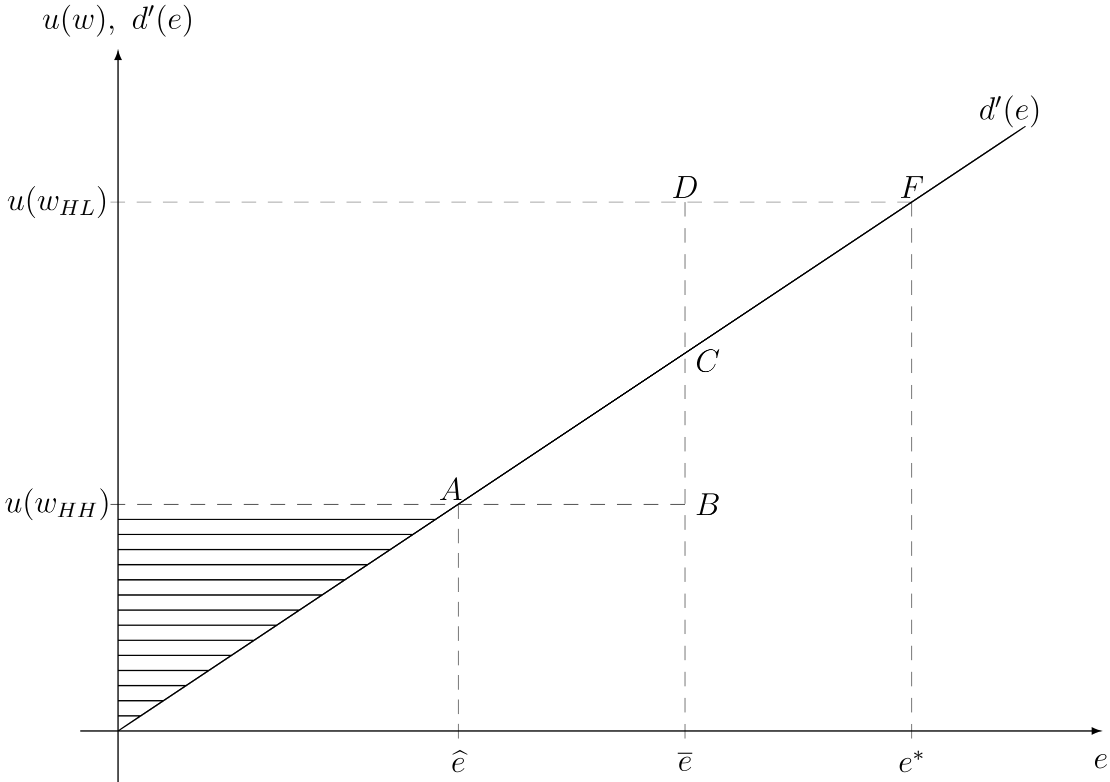
$$C'(e, \bar{e}) = v(d'(e)) + d''(e)(e - \bar{e}) \left\{ \frac{1}{u'(w_{HL})} - \frac{\bar{e}}{(\bar{e} - \hat{e})u'(w_{H:L})} \right\} \tag{8}$$

Note that first that the marginal cost of effort is *negative* for e sufficiently close to (but greater than) $\bar{e}$. To verify this, note that the negative term within curly brackets in the above expression tends to ∞ as $e \searrow \bar{e}$, since this implies $\hat{e} \nearrow \bar{e}$. Hence the cost of effort declines as effort is expanded beyond $\bar{e}$, while the marginal benefit to effort, which is $y_H - y_L$, is constant . It follows that it can never be optimal for a principal to choose to implement $\bar{e}$ if the other principal implements $\bar{e}$. Hence there cannot exist a symmetric equilibrium where the same effort level is implemented by both principals. In the single principal case, it will never be optimal to offer a symmetric contract.

Second, note that the term in curly brackets will be negative for any $e > \bar{e}$ provided that the agent is not too risk averse. Hence the marginal cost of effort will be lower than $v(d'(e))$. If the agent is risk neutral, so that v is the identity function, the marginal cost of effort reduces to

$$C'(e, \bar{e}) = d'(e) - d''(e)(e - \bar{e}) \frac{\hat{e}}{(\bar{e} - \hat{e})} \tag{9}$$

On the other hand, marginal cost of effort in the first best case (where effort is verifiable) equals $v'(d(e) + \bar{u})d'(e)$, which reduces to $d'(e)$ when the agent is risk neutral. Hence we find that the marginal cost of effort is lower with relative performance evaluation as compared to the first best, and there will be overprovision of effort.



Shaded area: rents for $\widehat{e}$, e^* Area ABC = Area CDF .

Figure 1: Rents and Effort

Some intuition for our results comes from Figure 1, where both the disutility of effort and the utility of the wage are plotted on the vertical axis, with effort on the horizontal axis. Suppose that the agent's outside option $\bar{u}$ is sufficiently low that the participation constraint is irrelevant, while limited liability constraints bite, so that $w_{LH} = w_{LL} = 0$. In order to induce any effort level less than $\bar{e}$, for example $\hat{e}$, the principal must set $u(w_{HH})$ equal to the marginal cost of effort at $\hat{e}$. Whereas the cost to the agent of this effort level is given by the area under the marginal cost curve, i.e. the area of the triangle, the benefit is given by the rectangle with sides $u(w_{HH})$ and $\hat{e}$. In consequence the agent earns an informational rent which is given by the area of the same rectangle above the marginal cost curve (minus the outside option $\bar{u}$). As long as $e \leq \bar{e}$, the rent that the principal must pay increases with the effort level e.

Now considering inducing an effort level $e^* > \bar{e}$. The principal must pay $u(w_{HL}) = d'(e^*)$ so that this effort level is locally incentive compatible. However, this implies that the agent earns a rent on effort levels above $\bar{e}$: whereas the additional benefit equals $u(w_{HL})(e^* - \bar{e})$, the additional cost to the agent is $d(e^*) - d(\bar{e})$. This additonal information rent is indicated by the shaded area. However, the principal can now reduce $u(w_{HH})$ below $d'(\bar{e})$, to $d'(\hat{e})$. The agent makes negative rents on additional effort levels in the range $[\hat{e}, \bar{e}]$, and $\hat{e}$ is chosen so that the value of these negative rents equal the positive rents on effort levels in the range $[\bar{e}, e^*]$. Hence the total informational rent earned by the agent at e^* is the same as that she earns at $\hat{e}$. Since $\hat{e}$ is decreasing in e^*, the informational rents earned by the agent *decline* with additional effort.

We conclude therefore that the principal will offer different contracts to the two identical agents. One of the agents will be given incentives to produce a relatively low level of effort, while the other agent will be made to choose high effort. It is of interest to note that the hard working agent will be worse off, i.e. her level of surplus will be lower than that of the agent who is made to choose low effort.

4 Conclusions

We conclude that limited liability plus the restriction to symmetric contracts offers one explanation for the absence of relative performance evaluation in agrarian contracts. Limited liability also explains why there is no informational advantage to a landlord from dividing up his plots between several tenants. If asymmetric contracts are permitted, then we see that they will be offered, and this is one reason for inequality between agents to increase for endogenous reasons.

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